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FREQUENCY CONVERSION IN GENERAL NONLINEAR MULTIPORT DEVICES

Vittorio RIZZOLI (1), Claudio CECCHETTI (2) and Alessandro LIPPARINI (1)

(1) Dipartimento di Elettronica, Informatica e Sistemistica, University of Bologna
(2) Fondazione Ugo Bordoni
Villa Griffone - 40044 Pontecchio Marconi - Bologna - ITALY

ABSTRACT

The conversion matrix of an arbitrary nonlinear multiport, operated in steady-state periodic regime, is defined and computed by a general straightforward approach. The results can be used to analyze the steady state for stability and generation of spurious, and to give a systematic solution to the generalized mixer problem. Some typical examples of application are presented and discussed.

INTRODUCTION

Modern computer-aided mixer analysis techniques rely upon the concept of admittance (or impedance) conversion matrix of a nonlinear device {1, 2}. Although several approaches to the computation of the conversion matrix of specific device topologies have been proposed in the technical literature {1 - 5}, a completely general method has not been available so far. This paper introduces an algorithm allowing a straightforward derivation of the conversion matrix of *any* nonlinear multiport for which an analytic time-domain or an equivalent-circuit description is available, irrespective of its topology. Some examples of application are presented.

The result of this work is of considerable importance: besides providing a key tool for the solution of the generalized mixer problem, it lends itself to impressive conceptual developments, such as general stability and noise analysis of pumped or self-oscillating nonlinear circuits.

GENERALIZED CONVERSION MATRIX

We consider a nonlinear device described by the set of time-domain equations

$$\underline{v}(t) = \underline{\phi}\left[\underline{x}(t), \frac{d\underline{x}}{dt}, \ldots \frac{d^n\underline{x}}{dt^n}\right]$$
$$\underline{i}(t) = \underline{\psi}\left[\underline{x}(t), \frac{d\underline{x}}{dt}, \ldots \frac{d^n\underline{x}}{dt^n}\right] \qquad (1)$$

where $\underline{v}$, $\underline{i}$ are vectors of voltages and currents at the device ports, and $\underline{x}$ is a vector of physical quantities used as state variables. For a well-behaved device all vectors in (1) have the same size n_D, equal to the number of device ports. With a suitable choice of both the device ports and the state variables, (1) can fit any device model, no matter how complicated.

Now we assume that the nonlinear device is connected with an n_D-port linear network and that the resulting circuit can support a periodic steady-state regime (either pumped or self-oscillating) with a fundamental frequency ω_0. Voltage and current harmonics of the steady state can be found by the harmonic-balance technique. If the equilibrium condition is perturbed by injecting a *small* signal $\exp(j\omega t)$, the resulting perturbation of the device state takes the form

$$\underline{\Delta x}(t) = \mathrm{Re}\left[\sum_{k=-\infty}^{\infty} \underline{\Delta X}_k \exp\{j(\omega + k\omega_0)t\}\right] \qquad (2)$$

where $\underline{\Delta X}_k$ is a vector (of size n_D) of complex spectral components in the k-th sideband. Similar expressions hold for the perturbations of voltages and currents at the device ports (with $\underline{\Delta X}_k$ replaced by $\underline{\Delta V}_k$, $\underline{\Delta I}_k$, respectively). The perturbed electrical regime must satisfy the device equations (1). Since the perturbation is small, these equations may be linearized around the periodic steady state, which results in a linear relationship among the sideband amplitudes $\underline{\Delta X}_k$, $\underline{\Delta V}_k$, $\underline{\Delta I}_k$. For ease of notation, we now introduce the vectors of all sideband amplitudes, namely $\underline{\Delta X}$, $\underline{\Delta V}$, $\underline{\Delta I}$ (i.e., $\underline{\Delta X}$ is the column of all subvectors $\underline{\Delta X}_k$ for $-\infty < k < \infty$). Thus we will have

$$\underline{\Delta V} = \underline{P}\ \underline{\Delta X}$$
$$\underline{\Delta I} = \underline{Q}\ \underline{\Delta X} \qquad (3)$$

from which

$$\underline{\Delta I} = \underline{Q}\ \underline{P}^{-1}\ \underline{\Delta V} = \underline{Y}_c\ \underline{\Delta V}$$
$$\underline{\Delta V} = \underline{P}\ \underline{Q}^{-1}\ \underline{\Delta I} = \underline{Z}_c\ \underline{\Delta I}\ . \qquad (4)$$

By definition $\underline{Y}_c$ ($\underline{Z}_c$) is the admittance (impedance) conversion matrix of the nonlinear device.

The matrices $\underline{P}$, $\underline{Q}$ can be computed by the following algorithm. First we derive from (1), and we evaluate in steady-state conditions ($\sim$), the ($n_D \times n_D$) Jacobian matrices

$$\left.\frac{\partial \underline{\phi}}{\partial \underline{y}_m}\right|_\sim = \sum_{p=-\infty}^{\infty} \underline{C}_{m,p} \exp(jp\omega_o t)$$
$$\left.\frac{\partial \underline{\psi}}{\partial \underline{y}_m}\right|_\sim = \sum_{p=-\infty}^{\infty} \underline{D}_{m,p} \exp(jp\omega_o t) \quad (5)$$

where $\underline{y}_o = \underline{x}$, $\underline{y}_m = d^m\underline{x}/dt^m$ ($m = 1, 2 .. n$). Since the steady state is periodic, so are the Jacobian matrices, which justifies the Fourier expansions on the right-hand side of (5). The coefficient matrices can actually be computed by the FFT. As a second step we introduce the ($n_D \times n_D$) matrices

$$\underline{P}_{k,p} = \sum_{m=0}^{n} \{j(\omega + k\omega_o)\}^m \underline{C}_{m,p}$$
$$\underline{Q}_{k,p} = \sum_{m=0}^{n} \{j(\omega + k\omega_o)\}^m \underline{D}_{m,p} \quad (6)$$

from which we finally get

$$\underline{P} \equiv \left[\underline{P}_{k,s-k}\right] \qquad \underline{Q} \equiv \left[\underline{Q}_{k,s-k}\right]. \quad (7)$$

In (7), s acts as the row index, and k as the column index of the generic ($n_D \times n_D$) submatrix ($-\infty < s < \infty$, $-\infty < k < \infty$). It has been verified that the usual conversion matrix of some elementary components, such as the nonlinear conductance and capacitance, can be reobtained from the above equations in a straightforward way.

STABILITY

As a first application we show that the generalized conversion matrix can be used to investigate the stability of the steady-state equilibrium condition in a simple, physically intuitive way. To do so, we consider a small injected signal of the form $\exp\{(\sigma + j\omega)t\}$ and look for the values of $(\sigma + j\omega)$ that represent the natural frequencies of the steady-state periodic regime. Since the perturbation is small, the nonlinear device equations still have the form (4), with ω replaced by $\omega - j\sigma$ in the expression of the conversion matrix. On the other hand, if the ($n_D \times n_D$) admittance matrix of the linear subnetwork is denoted by $\underline{Y}(\omega)$, the constraints imposed by the linear subnetwork on the sideband amplitudes may be written as

$$\underline{\Delta I} = - \underline{Y}_L \underline{\Delta V} \quad (8)$$

where

$$\underline{Y}_L = \text{diag}\left[\underline{Y}(\omega + k\omega_o - j\sigma)\right] \qquad -\infty < k < \infty . \quad (9)$$

Combining (9) and (4) yields the eigenvalue equation for the natural frequencies

$$\det(\underline{Y}_c + \underline{Y}_L) = 0 . \quad (10)$$

Note that (10) is formally identical, and conceptually similar, to the equation used to find the natural frequencies of a *linear* network. The corresponding quantities for the two cases are: the admittance conversion matrix (nonlinear case), and the conventional device admittance matrix (linear case); the diagonal sum (9) of the linear subnetwork admittances at all sidebands (nonlinear case), and the conventional linear subnetwork admittance matrix (linear case). For computational purposes, (10) by means of (3) is rewritten in the equivalent form

$$\det \underline{A} = 0 \quad (11)$$

where

$$\underline{A} \equiv \left[\underline{A}_{k,s-k}\right] \quad (12)$$

$$\underline{A}_{k,s-k} = \sum_{m=0}^{n} \{\sigma + j(\omega + k\omega_o)\}^m \underline{D}_{m,s-k} + $$
$$+ \underline{Y}(\omega + s\omega_o - j\sigma) \sum_{m=0}^{n} \{\sigma + j(\omega + k\omega_o)\}^m \underline{C}_{m,s-k} \quad (13)$$

($-\infty < s < \infty$, $-\infty < k < \infty$). The properties of the infinite matrix (12) are discussed in detail in ref. {6}, together with an algorithm for the numerical derivation of its Nyquist stability plot. This makes available a general tool for analyzing such effects as the generation of spurious tones in microwave oscillators, the existence of parametric instabilities in pumped-reactance circuits, and similar.

DIODE WITH NONLINEAR SERIES RESISTANCE

The equivalent circuit of a microwave diode is given in fig. 1. The (only) state variable used to describe the device is the voltage across the junction, indicated by x in the figure, according to (1). The diode model includes a nonlinear series resistance $R_S(x)$ depending on the junction voltage. In most previous approaches to mixer analysis, this nonlinear resistance has been replaced by a linear (constant) one, because its presence would considerably complicate the computation of the conversion matrix by conventional methods. On the other hand, the present theory can cope with the model of fig. 1 in a straightforward way.

Following the method outlined in section 2, we introduce the Fourier expansions

$$\left.\left[\frac{dR_s}{dx} i_D(x) + R_s(x) \frac{di_D}{dx}\right]\right|_\sim = \sum_{p=-\infty}^{\infty} \alpha_p \exp(jp\omega_o t) \quad (14a)$$

$$R_s\{\tilde{x}(t)\} \cdot C\{\tilde{x}(t)\} = \sum_{p=-\infty}^{\infty} \tau_p \exp(jp\omega_o t) \quad (14b)$$

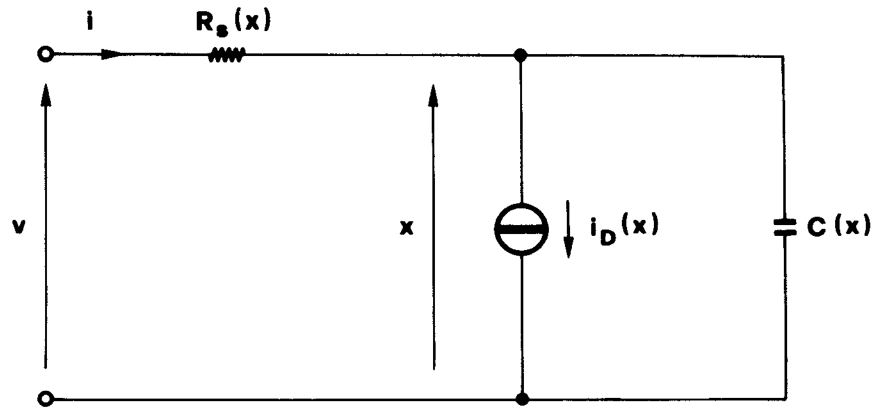

Fig. 1 - Nonlinear equivalent circuit of microwave diode.

$$\left.\frac{di_D}{dx}\right|_{\tilde{x}} = \sum_{p=-\infty}^{\infty} G_p \exp(jp\omega_o t) \qquad (14c)$$

$$C\{\tilde{x}(t)\} = \sum_{p=-\infty}^{\infty} C_p \exp(jp\omega_o t) \qquad (14d)$$

where $\tilde{x}(t)$ is the steady-state voltage across the diode junction. Next we define the matrices

$$\begin{aligned} \underline{\alpha} &\equiv \left[\alpha_{s-k}\right] \\ \underline{\tau} &\equiv \left[\tau_{s-k}\right] \\ \underline{G} &\equiv \left[G_{s-k}\right] \\ \underline{C} &\equiv \left[C_{s-k}\right] \\ \underline{\Omega} &= \mathrm{diag}\left[\omega + s\omega_o\right]. \end{aligned} \qquad (15)$$

The admittance conversion matrix of the diode is then given by

$$\underline{Y}_c = (\underline{G} + j\,\underline{\Omega}\,\underline{C}) \cdot (\underline{E} + \underline{\alpha} + j\,\underline{\Omega}\,\underline{\tau})^{-1} \qquad (16)$$

where $\underline{E}$ denotes the identity matrix.

NONCONVENTIONAL FET MODELS

Fig. 2 shows a nonlinear equivalent circuit of the Plessey GAT6 FET {7}. The intrinsic channel resistance is modeled as a nonlinear parametric resistor R_1 depending on gate voltage. Furthermore the circuit contains an internal feedback branch $R_f - C_f$ used to decouple the DC and AC behavior. Once again, this model would be very difficult to deal with by conventional methods. Note that a similar situation occurs when the equivalent circuit of the Gunn do

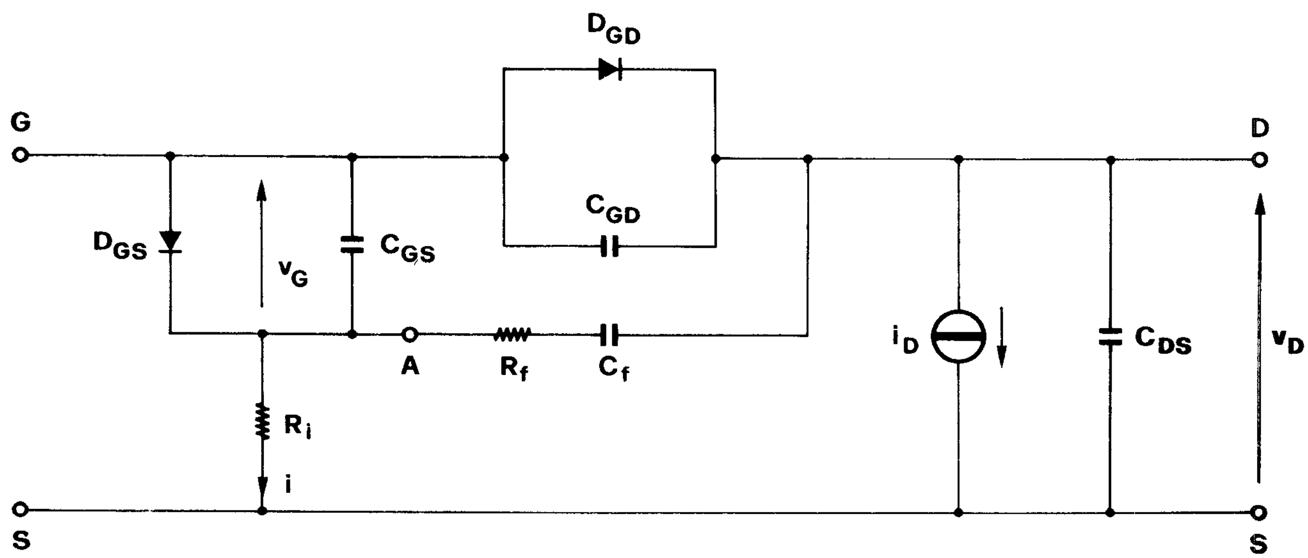

Fig. 2 - Nonlinear equivalent circuit of the Plessey GAT6 FET (after {7}).

main is included in the FET model.

To apply the approach described in this paper, we cut the circuit at point A and include the feedback branch (which has constant parameters) in the linear part of the network. We are thus left with a three-port nonlinear device (port nodes are G-S, D-S, A-S, respectively), for which we select as state variables the gate and drain voltages and the *current* through the channel resistance (namely i in fig. 2). This choice is essential in order to avoid numerical ill-conditioning because the resistance R_i takes very small values for certain gate voltages {7}. At this stage both the device equation (1) and the Jacobian matrices (5) can be written by inspection of fig. 2 in terms of v_G, v_D, i. The details of the derivation are not reported here for the sake of brevity.

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